Van der Waals Frictional Drag induced by Liquid Flow in Low- Dimensional Systems

A.I.Volokitin[1,2*] and B.N.J.Persson[1]
[1]Institut für Festkörperforschung, Forschungszentrum Jülich, D-52425, Germany
[2]Samara State Technical University, 443100 Samara, Russia

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Abstract

We study the van der Waals frictional drag force induced by liquid flow in low-dimensional systems (2D and 1D electron systems, and 2D and 1D channels with liquid). We find that for both 1D and 2D systems, the frictional drag force induced by liquid flow may be several orders of magnitude larger than the frictional drag induced by electronic current.

A great deal of attention has been devoted to the problem of frictional drag in low-dimensional systems [1, 2, 3, 4, 5, 6, 7] because of it importance for nanoscale detectors. Such detectors would be of great interest in micromechanical and biological applications [8, 9], where local dynamical effects are intensively studied.

A frictional drag will act on a 2D-electron system if an electric current flows in a second parallel 2D electron- system. This drag effect, suggested many years ago [10, 11], has been studied in two-dimensional quantum wells [6, 7]. Experiments [6] show that, at least for small separations, the friction

*Corresponding author. *E-mail address*:avoli@samgtu.ru

drag can be explained by the interaction between the electrons in the different layers via the fluctuating Coulomb field. However for large inter-layer separation the friction drag is dominated by phonon exchange [12].

Recently, flow of liquids over bundles of single-walled carbon nanotubes (SWNT) was found to generate a voltage in the sample in the flow direction [1, 2]. The dependence of the voltage on the flow speed was found to be logarithmic over five decades of variation of the speed. There have been attempts to explain this flow-induced voltage in electrokinetic terms, as a result of the streaming potential that develops along the flow of an electrolyte through a microporous insulator [13, 14]. Earlier Král and Shapiro [4] proposed that the liquid flow transfer momentum to the acoustic phonons of the nanotube, and that the resulting "phonon wind" drives an electric current in the nanotube. They also suggested, qualitatively, that the fluctuating Coulomb field of the ions in the liquid could drag directly the carriers in the nanotube. However, the first mechanism [4] requires an enormous pressure [2], while the second mechanism [4] result in a very small current, of order femtoAmperes [2]. In [2] another mechanism was proposed, which is related to the second idea of Ref. [4], but which requires neither localization of carriers nor drag at the same speed as the ions: Thermal fluctuations in the ionic charge density in the fluid near the nanotube produce a stochastic Coulomb field acting on the carries in the nanotube. The fluctuation-dissipation theorem tells us that the zero-frequency friction coefficient is proportional to the time integral of the force-force correlation function, where the total force is determined by summation of all the forces on all charged carries in the nanotube. In [2] it was assumed that the forces on the different charged carriers are uncorrelate, which is justified only for very low carrier density. In fact, [2] considered the friction between a moving point charge and the surrounded medium. An external charge will induce an "image" charge in the surrounded medium. Because of the finite response time this "image" charge lag behind the moving charge, which results in a force acting on the charge, referred to as the *electrostatic* friction. For neutral systems, such as a nanotube, the *electrostatic* friction proposed in [2] will vanishing.

In [5] it was assumed that the liquid molecules nearest to the nanotube form a 2D-solidlike monolayer, pinned to the nanotube by adsorbed ions. As the liquid flow, the adsorbed solid monolayer performs stick-slip type of sliding motion along the nanotube. The drifting adsorbed ions will produce a voltage in the nanotube through electronic friction against free electrons inside the nanotube. As in [4] it was assumed that the drift velocity of the

electrons in the nanotubes is equal to the liquid flow velocity.

In [3] a model calculations of the frictional drag were presented involving a channel containing overdamped Brownian particles. The channel was imbedded in a wide chamber containing the same type of Brownian particles with drift velocity parallel to the channel. It was found that the flow of particles in the chamber induces a drift of the particles in the channel.

In this Letter we will address the intriguing idea of using frictional drag as a noncontact means to detect motion in surrounded liquid. We present a frictional drag theory based on the theory of the van der Waals friction [15, 17, 16]. The origin of the van der Waals friction is connected with the fluctuating electromagnetic field which is always present outside of any medium due to thermal and quantum fluctuations of the charge density inside the medium. This fluctuating electromagnetic field induces polarization of the medium, and is responsible for many important phenomena such as radiative heat transfer and the van der Waals interaction [16]. When two media are in relative motion, the induced polarization will lag behind the fluctuating polarization inducing it, and this gives rise to the so-called van der Waals friction.

The origin of the van der Waals friction is closely connected with the Doppler effect. Let us consider two flat parallel surfaces, separated by a sufficiently wide insulator gap, which prevents particles from tunneling across it. If the charge carriers inside the volumes restricted by these surfaces are in relative motion (velocity v) a frictional stress will act between surfaces. This frictional stress is related with an asymmetry of the reflection amplitude along the direction of motion; see Fig. 1. If one body emits radiation, then in the rest reference frame of the second body these waves are Doppler shifted which will result in different reflection amplitudes. The same is true for radiation emitted by the second body. The exchange of "Doppler-shifted-photons" will result in momentum transfer and to the van der Waals friction. According to [15, 17], the frictional stress between two flat parallel surfaces at separation $d \ll \lambda_T = c\hbar/k_BT$ is determined by

$$\sigma_{\parallel} = \frac{\hbar}{2\pi^3}\int_{-\infty}^{\infty} dq_y \int_0^{\infty} dq_x q_x e^{-2qd}\left\{\int_0^{\infty} d\omega[n(\omega) - n(\omega + q_x v)]\right.$$

$$\left.\times\left(\frac{\mathrm{Im}R_{1p}(\omega + q_x v)\mathrm{Im}R_{2p}(\omega)}{\mid 1 - e^{-2qd}R_{1p}(\omega + q_x v)R_{2p}(\omega)\mid^2} + (1 \leftrightarrow 2)\right)\right.$$

$$-\int_0^{q_x v} d\omega[n(\omega)+1/2]\left(\frac{\mathrm{Im}R_{1p}(\omega-q_x v)\mathrm{Im}R_{2p}(\omega)}{\mid 1-e^{-2qd}R_{1p}(\omega-q_x v)R_{2p}(\omega)\mid^2}+(1\leftrightarrow 2)\right)\Bigg\}. \tag{1}$$

where $n(\omega)=[\exp(\hbar\omega/k_BT-1]^{-1}$ and $(1\leftrightarrow 2)$ denotes the term which is obtained from the first one by permutation of indexes 1 and 2. R_{ip} $(i=1,2)$ is the reflection amplitude for surface i for p -polarized electromagnetic waves. The reflection amplitudes for a 2D-electron system are determined by [17]

$$R_{ip}=\frac{\epsilon_{ip}-1}{\epsilon_{ip}+1}, \tag{2}$$

where $\epsilon_{ip}=4\pi iq\sigma_i(\omega,q)/\omega\varepsilon+1$, σ_i is the longitudinal conductivities of the layer i and ε is the dielectric constant of the surrounded dielectric. The longitudinal conductivity can be written in the form $\sigma_l(\omega,q)=-\mathrm{i}\omega\chi_l(\omega,q)/q^2$, where χ_l is the finite lifetime generalization of the longitudinal Lindhard response function for 2D-electron gas [18, 19]. The friction force per unit charge in the layer is determined by $E=\sigma_\parallel/n_se$, where n_s is the 2D-electron concentration in the layer. For $v\ll v_F$, where v_F is the Fermi velocity, the friction force depends linearly on velocity v. For $d=175$ Å at $T=3$ K, and with $n_s=1.5\times10^{15}$ m^{-2}, the electron effective mass $m^*=0.067$ m$_e$, $v_F=1.6\times10^7$ cm/s, the electron mean free path $l=v_F\tau=1.21\times10^5$ Å, and $\varepsilon=10$ (which corresponds to the condition of the experiment [6]) we get $E=6.5\times10^{-6}v$ V/m, where the velocity v is in m/s. For a current 200 nA in a two-dimensional layer with the width $w=20\mu$m the drift of electrons (drift velocity $v=60$m/s) creates a frictional drag force per unit charge in the adjacent quantum well $E=4\cdot10^{-4}$V/m. Note that for the electron systems the frictional drag force decreases when the electron concentration increases. For a example, for 2D-quantum wells with high electron density ($n_s=10^{19}$ m^{-2}, $T=273$ K, $\tau=4\times10^{-14}$ s, $\varepsilon=10$, $m^*=m_e$) at $d=175$ Å we get $E=1.2\times10^{-9}v$ V/m.

Let us replace one 2D-layer by semiinfinite chamber with liquid containing ions. The reflection amplitude for the interface between dielectric and liquid is given by Eq. (2), where

$$\epsilon=\frac{\varepsilon_0\lambda(Dq_D^2-i\omega)}{\varepsilon(Dq_D^2q-i\omega\lambda)}, \tag{3}$$

where D is the diffusion coefficient of the ions, ε_0 is the dielectric function of liquid without ions, $q_D=\sqrt{4\pi ne^2/\varepsilon_0k_BT}$ is the Debye screening wave

number, $\lambda = \sqrt{q^2 + q_D^2 - i\omega/D}$, n is the ion concentration. In this case, for $v \ll v_F$ the friction force also depends linearly on velocity v. In particular, for $n = 10^{24}$ m^{-3}, $T = 273$ K, $\varepsilon_0 = 80$, $D = 10^{-9}$m^2/s for high electron density ($n_s = 10^{19}$m^{-2}) for 2D-electron system we get $E = 1.4 \times 10^{-6} v$ V/m. This friction is three orders of magnitude larger than the friction between the 2D-electron system with high electron concentration, and of the same order of magnitude as the friction between 2D-electron systems with low electron concentration.

Let us replace the second 2D-layer by narrow 2D-channel with thickness d_c. For $q_D d_c \ll 1$ the flowing liquid in the channel can be considered as a 2D-liquid. The conductivity of a 2D-liquid with ions performing Brownian motion is given by

$$\sigma(\omega, q) = -\frac{i\omega d_c}{4\pi}\left(-1 + \varepsilon_c\left(1 + \frac{q_{Dc}^2}{q^2 - i\omega/D_c}\right)\right) \tag{4}$$

where ε_c, D_c and q_{Dc} are the dielectric function, the diffusion coefficient and the Debye wave number for the liquid in the channel, respectively. Fig. 3 shows the dependence of the frictional drag force per unit charge on the liquid flow velocity for identical liquid in the channel and in the semi-infinite chamber with the same parameters as used above for the liquid and the separation between the channel and chamber $d = 1$nm. The frictional drag force on the ions in the channel initially increases with the flow velocity, reaches a maximum and then decreases at large value of the flow velocity, in agreement with the model calculation in [3]. The position of maximum decreases when the density of ions decreases. The frictional drag force induced by liquid flow in the narrow channel is nine orders of magnitude larger than the friction force induced in a 2D-electron system.

As a limiting case of the situation considered above, let us consider a 2D-system immersed in a flowing liquid in an infinite chamber. We assume that the liquid flows along the x-axis, and that the plane of 2D-system coincides with the xy- plane. According to the fluctuation-dissipation theorem the average value of the correlation function for the Fourier components of the Coulombic potential for a infinite medium in the plane of 2D-system is determined by [16]

$$< \varphi^f(\omega, q, 0)\varphi^{*f}(\omega, q, 0) >= 4\hbar(n(\omega) + 1/2)\mathrm{Im}\Sigma(\omega, q, 0), \tag{5}$$

where

$$\Sigma(\omega, q, 0), = -\int_{-\infty}^{\infty} \frac{dk_z}{2\pi}\frac{1}{k^2\varepsilon_0\varepsilon(k)}, \tag{6}$$

where $\mathbf{k} = (\mathbf{q}, k_z)$. The dielectric function of the Debye plasma is determined by

$$\varepsilon(k) = 1 + \frac{q_D^2}{k^2 - i\omega/D} \tag{7}$$

According to the fluctuation-dissipation theorem the average value of the correlation function for the Fourier components of the fluctuating current density in 2D-system is determined by [16]

$$< j_q^f(\omega, q) j_q^{f*}(\omega, q) >= \frac{\hbar q^2}{\pi\omega}(n(\omega) + 1/2)\mathrm{Re}\sigma(\omega, q). \tag{8}$$

The friction force per unit area of 2D-system is given by

$$\gamma_{\parallel} = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{q_x}{q} < E_q(\omega, q, 0)\rho^*(\omega, q) > \tag{9}$$

where $E_q = -iq\varphi_q = E_q^f + E_q^{ind}$, $\rho = \rho^f + \rho^{ind}$. The fluctuating electric field E_q^f is the sum of the fluctuating field which exists in the infinite medium without the 2D-system, and the electric field created by the fluctuating charge density $\rho_q^f = qj_q^f/\omega$ in presence of the flowing liquid. The induced electric field E_q^{ind} is created by the charge density $\rho_q^{ind} = qj_q^{ind}/\omega$ induced in 2D-system by the electric field E_q. The correlation functions (5) and (8) are determined in the rest reference frames of the liquid, and 2D-system, respectively. To find the relation between electric fields in the different reference frame we use the Galilean transformation, what will give rise to Doppler shift of the frequencies of the electric fields in the different reference frames. Solving the Poisson's equation with the fluctuating charge density ρ^f, and the charge density induced by the fluctuating electric field $E_q^f = -iq\varphi^f$ as the sources of the field we get E_q and from Ohm low we get ρ. The result of these calculations is:

$$\sigma_{\parallel} = \frac{\hbar}{2\pi^2}\int_{-\infty}^{\infty} dq_y \int_0^{\infty} dq_x q_x q^2 \Big\{ \int_0^{\infty} d\omega [n(\omega) - n(\omega + q_x v)]$$

$$\times \left(\frac{\mathrm{Re}\sigma(\omega + q_x v)\mathrm{Im}\Sigma(\omega)}{(\omega + q_x v) \mid 1 - 4\pi i q^2 \sigma(\omega + q_x v)\Sigma(\omega)/(\omega + q_x v) \mid^2} + (\omega + q_x v \leftrightarrow \omega) \right)$$

$$- \int_0^{q_x v} d\omega [n(\omega) + 1/2] \Big(\frac{\mathrm{Re}\sigma(\omega - q_x v)\mathrm{Im}\Sigma(\omega)}{(\omega + q_x v) \mid 1 - 4\pi i q^2 \sigma(\omega - q_x v)\Sigma(\omega)/(\omega - q_x v) \mid^2}$$

$$+ (\omega + q_x v \leftrightarrow \omega) \Big) \Big\}. \tag{10}$$

where $(\omega + q_x v \leftrightarrow \omega)$ denotes the term which is obtained from the first one by permutations of the arguments $\omega + q_x v$ and ω. With the same parameters as used above for the liquid, and for the high density 2D-electron system, we get $E = 8.1 \cdot 10^{-6} v$ V/m. For a 1D-electron system we obtained a formula which is similar to Eq. (10). Fig. 2 shows the result of the calculations of the friction drag force (per unit charge) for a 1D-electron system with the electron density per unit length $n_l = 3 \times 10^9 \mathrm{m}^{-1}$, the temperature $T = 300$ K, and with the same parameters for the liquid as used above. For the 1D-electron system we obtained a slight deviation from the linear dependence of the frictional drag on the liquid flow velocity. The frictional drag for the 1D-electron system is one order of magnitude larger than for the 2D-electron system. Fig. 4 shows the dependence of the frictional drag force per unit charge in the the 2D-channel with liquid on the liquid flow velocity in the infinite chamber assuming identical liquid in the channel and in the chamber. In this case the maximum in the frictional drag force is larger, and the decay at large velocities is slower in comparison with the semi-infinite chamber at separation $d = 1$nm. Qualitatively, the same results we obtained for a 1D-channel. For a channel with open ends the frictional drag force will induce a drift motion of the ions in the liquid with velocity $v_d = D_c e E / k_B T$. The positive and negative ions will drift in the same direction. In contrast to electron systems, for the channel with closed ends, in the case of equal concentration of the positive and negative ions, the frictional drag will not induce a voltage because displacement of the ions under the action of the frictional drag force will not violate charge neutrality of the liquid. However, the frictional drag will induce a pressure difference $\Delta p = nLeE$, where L is the length of the channel. For example, if $n = 10^{24} \mathrm{m}^{-3}$, $L = 100 \mu$m and $E = 1000$V/m we get pressure difference $\Delta p = 10^4$ Pa, which should be easy to measure.

In this Letter we have shown that the van der Waals frictional drag force induced in low-dimensional system by liquid flow can be several orders of magnitude larger than the friction induced by electron current. For a narrow 2D-channel with liquid the frictional drag force is several orders of magnitude larger than for 2D-electron systems. In the contrast to 2D-electron system the frictional drag force for a narrow channel depends nonlinearly on the flow velocity. These results should have a broad application for studying of the van der Waals friction and in the design of nanosensors.

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FIGURE CAPTIONS

Fig. 1. Two bodies moving relative to each other will experience van der Waals friction due to Doppler shift of the electromagnetic waves emitted by them.

Fig. 2. The frictional drag force per unit charge in a 1D-electron system induced by liquid flow in infinite chamber as a function of the flow velocity. The temperature $T = 300$ K, the ion concentration in liquid $n = 10^{24}\mathrm{m}^{-3}$, the diffusion coefficients of ions $D = 10^{-9}\mathrm{m}^2/\mathrm{s}$, the dielectric constant of the liquid $\varepsilon_0 = 80$, the electron concentration per unit length $n_l = 3 \cdot 10^9\mathrm{m}^{-1}$, the electron relaxation time $\tau = 4 \cdot 10^{-14}\mathrm{s}$

Fig. 3. The frictional drag force per unit charge for ions in a 2D-channel with liquid induced by liquid flow in semi-infinite chamber as a function of the flow velocity for identical liquids in the channel and chamber. The temperature $T = 300$ K, the ion concentration in liquid $n = 10^{24}\mathrm{m}^{-3}$, the diffusion coefficients of ions $D = 10^{-9}\mathrm{m}^2/\mathrm{s}$, the dielectric constant of the liquid $\varepsilon_0 = 80$, the separation between the channel and semi-infinite chamber $d = 1\mathrm{nm}$.

Fig. 4. The same as Fig. 4 but for infinite chamber.

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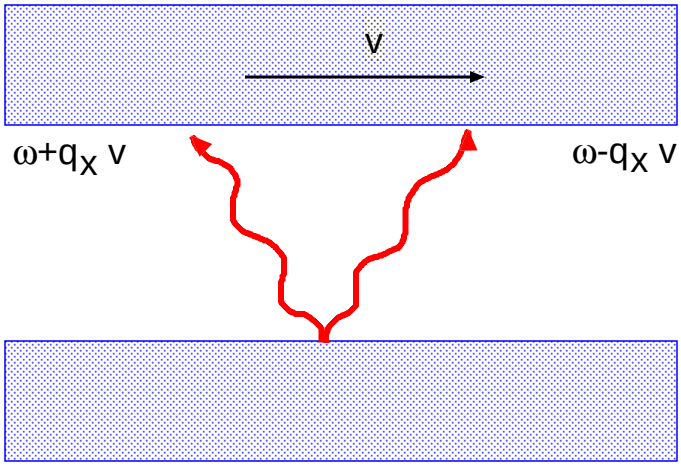
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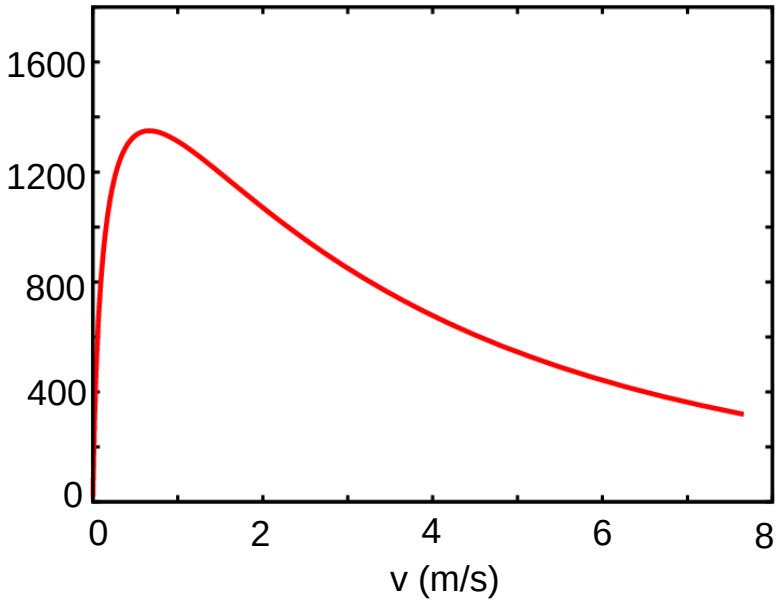
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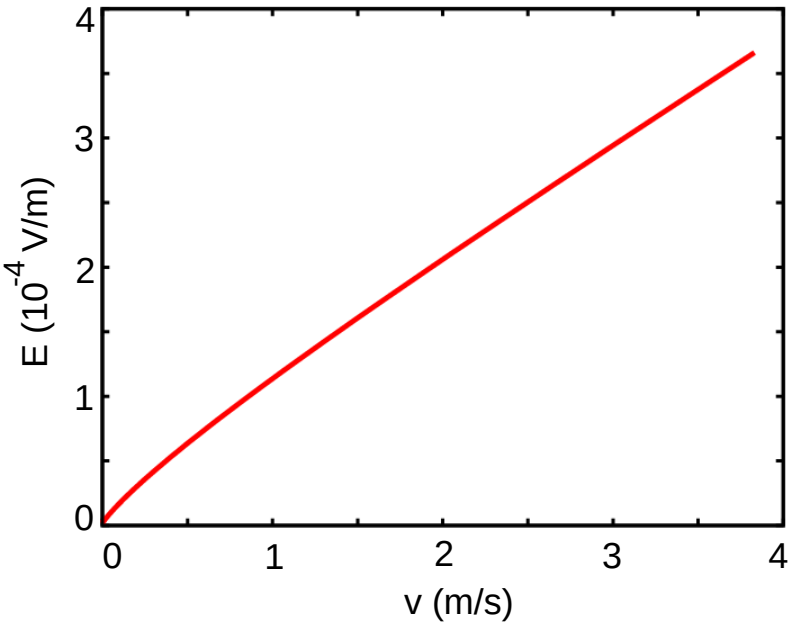
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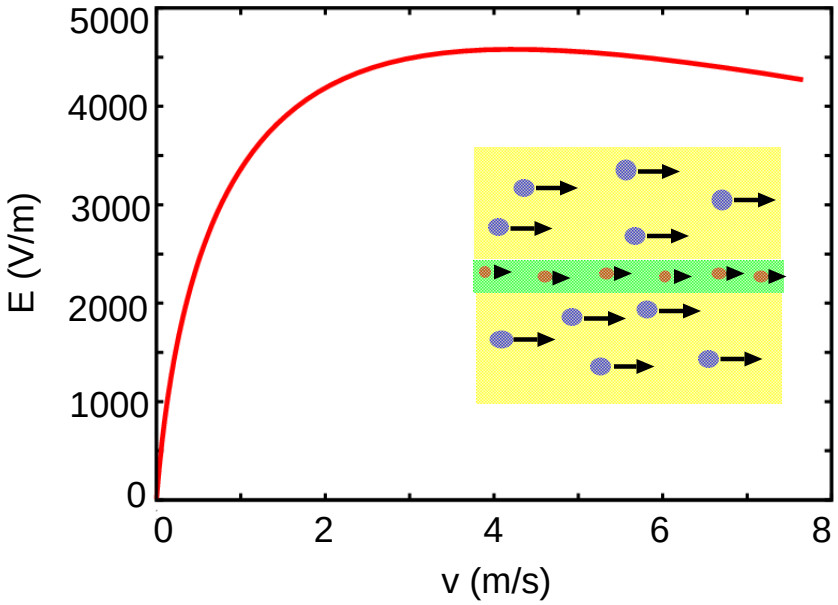
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4000
3000
2000
1000
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E (V/m)
0
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6
8
v (m/s)